Reg. No. : | | | | | | | | | | | |

Question Paper Code : 61280

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2014.

First Semester

Civil Engineering

MA 1101 — MATHEMATICS – I

(Common to all branches)

(Regulation 2008)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the sum of the squares of the eigenvalues of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.

2. Find the associated matrix related to the quadratic form $3x_1^2 - 7x_3^2 + x_1x_2 + 12x_2x_3$.

3. Find k if $\frac{x-2}{3} = \frac{y-1}{2} = \frac{z-3}{k}$ and $\frac{x-3}{k} = \frac{y-2}{3} = \frac{z-4}{5}$ are coplanar.

4. Find the length of the tangent drawn from $(1,2,3)$ to the sphere $x^2 + y^2 + z^2 - 3x + 4y - 5z + 7 = 0$.

5. Find the radius of curvature of $y = e^{\sqrt{3x}}$ at $x = 0$.

6. Find the envelope of the family of lines $\frac{x}{t} + yt = 2c$, t being parameter.

7. Find the nature of the stationary point $(1,2)$ of the function $f(x,y)$ which has $f_{xx} = 6x, f_{xy} = 0, f_{yy} = 6y$.

8. If $x = u - uv, y = uv$, find $\frac{\partial(u,v)}{\partial(x,y)}$.

9. Find the particular integral of $(D^2 - 2D + 5)y = e^{-x} \sin 2x$.

10. Convert $(x^2D^2 - xD - 3)y = \frac{1}{x}\cos(2\log x)$ into differential equation with constant coefficients.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and eigenvectors of the adjoint matrix A, given that $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$. (8)

(ii) Diagonalise $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$ by an orthogonal transformation. (8)

Or

(b) (i) Verify that the eigenvectors of a real symmetric matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ are orthogonal pairs. (8)

(ii) Verify Cayley–Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ and hence the inverse of A. (8)

12. (a) (i) Find the length and equations of the shortest distance between the lines $\frac{x-1}{1} = \frac{y-2}{-2} = \frac{z-3}{3}$ and $\frac{x+1}{2} = \frac{y}{-1} = \frac{z-1}{3}$. (8)

(ii) Find the equation of the right circular cylinder whose axis is the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$ and whose radius is equal to 5. (8)

Or

(b) (i) Find the equation of the sphere passing through the circle $x^2 + y^2 + z^2 - 2x - 3y + 4z + 8 = 0$, $3x + 4y - 5z = 3$ and having its centre on the plane $4x - 5y - z = 3$. (8)

(ii) Find the equation of the cone whose vertex is $(3,1,2)$ and base curve is $2x^2 + 3y^2 = 1, z = 1$. (8)

13. (a) (i) Show that the curves $y = c\cosh\frac{x}{c}$ and $x^2 = 2c(y-c)$ have the same curvature at the points where they cross the y-axis. (8)

(ii) Find the evolute of the parabola $y^2 = 4ax$, considering it as envelope of its normals. (8)

Or

(b) (i) Find the evolute of the cycloid $x = a(\theta + \sin\theta), y = a(1-\cos\theta)$. (8)

(ii) Find the envelope of the family of lines $x\cos\alpha + y\sin\alpha = c\sin\alpha\cos\alpha$, α being parameter. (8)

14. (a) (i) If $z = f(u,v)$, where $u = lx + my$ and $v = ly - mx$, prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 4(l^2 + m^2)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$. (8)

(ii) Find the Taylor series expansion of $e^x \cos y$ about $(0,0)$ up to the terms of the third degree. (8)

Or

(b) (i) If $u=xyz$, $v=xy+yz+zx$ and $w=x+y+z$, find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$. (6)

(ii) Discuss the maxima and minima of $f(x,y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$. (10)

15. (a) (i) Solve $(D^2 + 2D - 1)y = (x + e^x)^2$. (8)

(ii) Solve the simultaneous equations $(D+4)x + 3y = t$, $2x + (D+5)y = e^{2t}$. (8)

Or

(b) (i) Solve $((3x+2)^2 D^2 + 3(3x+2)D - 36)y = 3x^2 + 4x + 1$. (8)

(ii) Solve $(D^2+1)y = x\cos x$ by method of variation of parameters. (8)